

Ordinary Geometry

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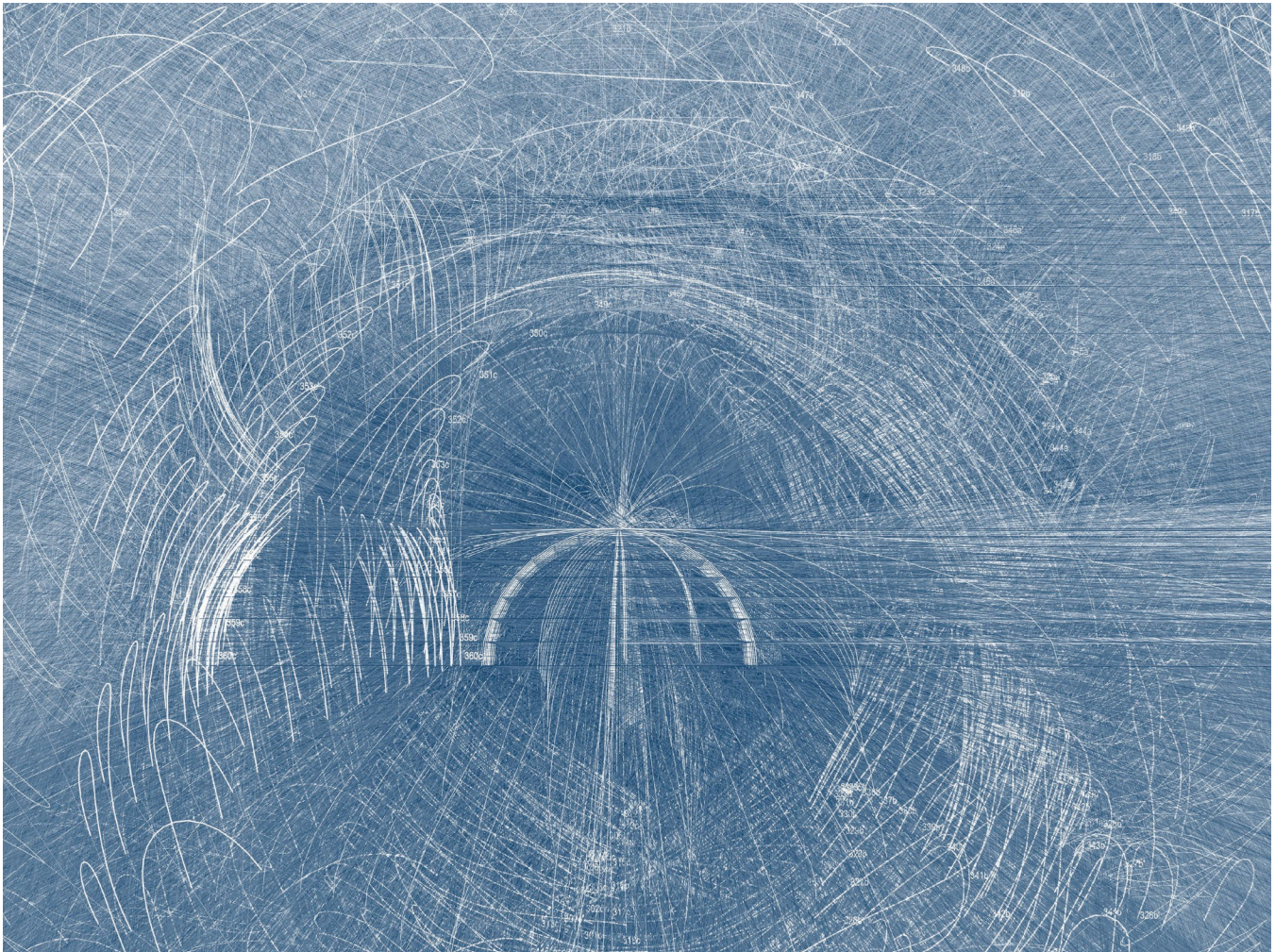


Figure 1. The two dimensional orthographic projection of a semi-circle. Image by Author.

The relevance of drawing in a discipline immersed in digital technology is increasingly under scrutiny. At the center of this problem is architecture's core representational strategy, orthographic projection. However, if instead of considering it as method for the projection of views it is understood as a series of operations used to calculate form independent of visualization the problem is different. It becomes a problem of translation in lieu of simulation. Rules initially used to govern the relationship of marks on a piece of paper or stone can be used to govern relationships in a digital environment, producing distinct images independent of references to historic media or no images

at all. This paper argues for the projective role of history in the implementation of technology. It focuses on the most ordinary type of architectural drawing, orthographic projection, and speculates on its potential to be re-imagined as a digital process. It begins with the reconstruction and formal analysis of the orthographic drawings of the Italian architect Guarino Guarini (1624-1683) and concludes with the translation of his techniques into digitally animated drawings. In lieu of understanding the development of architectural drawing and geometry as one of linear progress, it argues for the projective role of history in the development of architectural technologies.

ORDINARY PROJECTION

Notice that I have been able to describe the fantastic worlds above imagined without ceasing to employ the language of ordinary geometry.

—Henri Poincaré, *Science and Hypothesis*

Projection has been incorporated into so many electronic and mechanical processes that it no longer needs much space in our imagination. We do not normally have to think spatial relations out this way, and there seems little point in making anyone do so when it can be done instantaneously with such exactitude and facility in a black box.

—Robin Evans, *The Projective Cast: Architecture and Its Three Geometries*

In architecture, there is no more ordinary drawing than an orthographic projection. It has existed at least since the 15th century, becoming the definitive geometry of drawing in architecture in the early 16th century. It is the default mode of representation to describe the shape of a building in both built and unbuilt works of architecture, reducing the complexity of a four-hundred year old drawing process to the production of three views: plan, section, and elevation. Most architectural software includes a tool to produce simulated orthographic drawings from a digital model in the form of views. Even within recent critical analysis, orthographic projection is limited to the discussion of views. However, in the development of orthographic projection, a variety of techniques were created that did not deal with the production of plan, section, and elevation. In some extreme cases the drawings were used as graphic calculators, providing only distance without form. These drawings, tied to the production of stone cutting templates, present a divergent history of architectural drawing. A history of architectural drawing in which the computation of form was independent of its visualization. A history of drawing form without seeing it.

Orthographic projection is at once a precise geometric term and a problematic architectural one. It is defined as the geometric operation that translates a point from one position in space onto a perpendicular plane. Most textbooks on computational geometry provide a mathematical definition of the projective process and an algorithm. The algorithm could be used to return a set of coordinates indicating the point's new position, or it could be tied to a set of graphical objects that simulate the process on the computer screen. Orthographic projection is therefore a geometric operation that is independent of its visual representation. In architecture, the term has a different significance. It is associated with architectural drawings, and therefore with the representation of architecture as lines on a piece of paper. This association places it in both an important position and a tenuous one. Some argue for

the continued importance of orthographic projection as the means by which architecture is designed and communicated, and others see it as a vestige of an old technology (drawing on paper) that will soon be obsolete. It is represented in drawings and images, but it is not limited by either. The process was developed from principles set forth in Euclid's *Elements* (300 B.C.E). *Elements* is an unillustrated text that does not deal with drawing. Instead it uses words in the form of logical argument to prove geometric relationships. Orthography may be associated with drawing, but its operations are defined by words. It developed through the combination and adaptation of Euclidean principles to solve three-dimensional problems on a two-dimensional surface: the floor of a cathedral, a stone, or a sheet of paper. It originated as a drawing practice but its relevancy to architecture is elsewhere.

COMPUTING STONE

The use of orthographic projection to define the shape of stone vault construction has been termed stereotomy since the 17th century. This classification was probably first used in print by the stone mason Jaques Curabelle in 1644, but had a much earlier point of origin. Both prior to and after Curabelle's use of the term, stereotomy was referred to with multiple names including the art of lines and orthographic projection. Even within discussions of stereotomy, scholars still debate whether it is drawing practice or fabrication practice. Robin Evans's important analysis of Philibert de Lorme indicates that de Lorme's treatise, *Premier tome de l'architecture* (1561) was the first publication to specifically deal with the subject. While de Lorme, may have produced the first treatise, he is not the inventor of stereotomy or orthographic projection. Instead, he is credited with both documenting the techniques of gothic stone cutting and of adapting them to the production of classical forms. A large number of treatises were produced on stereotomy between the time of de Lorme's first publication and Gaspard Monge's codification of descriptive geometry in 1795. Nonetheless, Evans is careful to point out that only one other "well-known architect" developed a theory of stereotomy and orthographic projection in a treatise besides de Lorme, Guarino Guarini.

Guarino Guarini is the architect of significant works of religious architecture exemplified in San Lorenzo (1687) and Santissima Sindone (1694) in Turin, Italy. His treatise on architecture, *Architettura civile* (1737), was published long after his death in 1683 and completed by Bernardo Vittone from an incomplete manuscript. The tractate on stereotomy, "Dell' Ortografia Gettata," deals with the distortion of the semicircle through orthographic projection, and the subsequent development of vaults and stone cutting templates. Within Guarini's treatise there are a number of practices that present drawings as the computation of form in lieu of its representation. An example is the construction of drawings for non-spherical domes. Spherical domes contain circular sections that are identical through both ninety and one-hundred

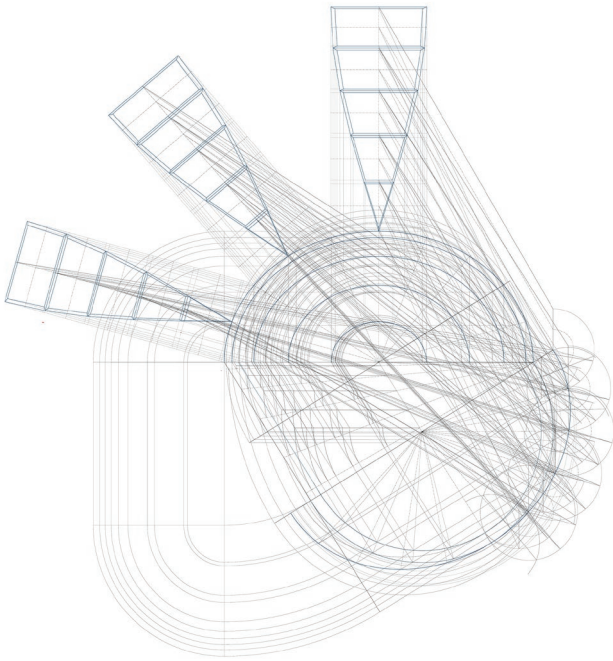


Figure 2. Lastra 14, Figura 3, Tractato 4. The reconstruction of Guarini's projection of an ellipsoidal dome. The triangular space between the two sets of projected ellipses is used to insert Euclid's Intercept theorem into an orthographic projection. Image by Author.

and eighty degrees of rotation. This allows section views to be orthographically constructed from front views without additional information. The plan and section of a sphere are the same. In contrast, ellipsoid, hyperboloid, paraboloid, and ovoid vaults do not share this characteristic. The section in each of these forms varies through ninety degrees of rotation. Stated differently, an ellipsoid cannot be generated from an ellipse through orthographic views alone. An additional means of drawing is required. To solve this Guarini used Euclid's intercept theorem to construct a drawing that calculated variable curvature through proportional relationships.

In describing the process of delineating an ellipsoid vault, Guarini refers to Euclid both directly and indirectly. He begins by directly citing tractate twenty-five and proposition eleven from his own treatise on Euclidean geometry in which he describes the proportional relationship between parallel sections through an ellipsoid. Next he solves the problem using Euclid's intercept theorem, without directly referencing it. In book six of *Elements*, proposition two, Euclid states, "If a straight line be drawn parallel to one of the sides of a triangle, it will cut the sides of the triangle proportionally." This statement, absent of reference to ellipses, is the basis of the Guarini's drawing. The proposition, described as the "intercept theorem" establishes a proportional relationship between non-parallel lines via the use of a parallel projectors. It establishes a process, in which any set of two or more lines can be proportionally divided. Guarini uses this process to establish a set of intersecting lines composed of the major

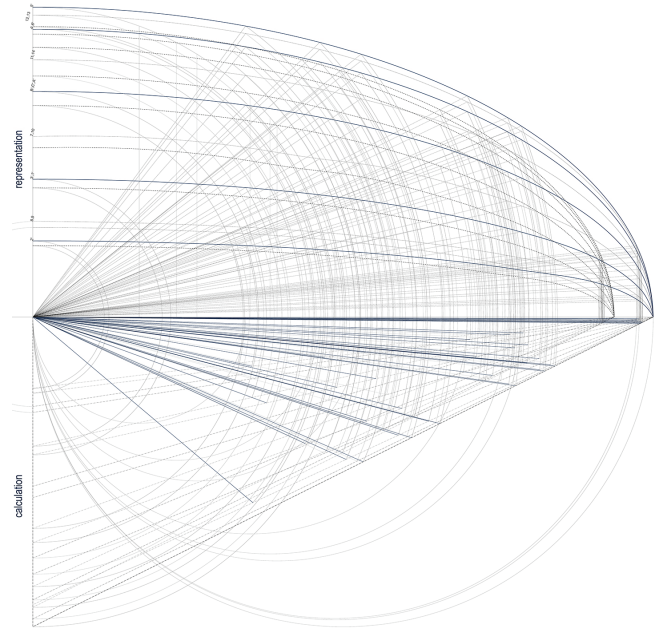


Figure 3. The section profiles of an elliptical dome generated by Euclid's Intercept Theorem after Guarini. The underside of the drawing calculates the form, and the top translates it into a polar grid. Image by Author

and minor diameter of the generating ellipse. Once parallel lines are added at intervals equivalent to a specific subdivision of the ellipse, it is a short step to deriving the true dimensions of a set of parallel sections through the ellipsoid vault (figure 2).

Guarini's drawing was a graphical calculation that did not directly produce a representation of shape. Instead it produced a grid with one set of parallel lines and one set of lines that converge at a point. The intersections of the lines provided the distance from a center to a point on the elliptical section. The points were not ordered based on the shape and no figure representing a view of the curve could be traced inside of the grid. The information had to be measured and then transferred onto a polar grid to describe the curvature (figure 3). The description of curvature through orthographic projection, was contingent upon the products of a drawing that computed form without reference to perceptual space. Orthography, when understood through its historical manifestations, presents an alternative form of drawing, in which the outcome is not an abstracted view of an object, but rather the computation of form.

Guarini's treatise was completed by Bernardo Vittone long after his death. It is not clear if the stereotomic drawings contained in "Dell' Ortografia Gettata," are Guarini's or are Vittone's interpretation of his text. Werner Müller argued that geometric errors present in the drawings are evidence of Vittone's inexperience with stereotomy, and may prove that the drawings are in fact his and not Guarini's. In this manner the drawings contained in

"Dell' Ortografia Gettata," can be seen as the graphic translation of the procedures set out in Guarini's manuscript. The following sections explain another translation of "Della Ortografia Gettata." The process began with the analysis and reconstruction of Guarini's drawings. It then shifted to translating Guarini's observations into sets of instructions through the use of a symbolic language (code) that can be repeated overtime with different variables. All of the animations are two dimensional (figure 1). All calculation is confined to operations on a two dimensional plane. There are no surfaces, three-dimensional objects, or three-value coordinates. The animations are computational not because they are created with the aid of a computer, but because Guarini's methods, from which they are derived, are procedural, defined by variable inputs, and produce a graphic output that is a representation not of space but of calculation. The animations are two-dimensional orthographic projections of ordinary geometry.

INVARIANT VARIABLES

Guarini demonstrated that parallel sections through an ellipsoid vault could be generated through the use of the Euclid's intercept theorem. The sections vary, but because they are parallel to one another they are proportional variations of the same invariant type, an ellipse. Guarini capitalizes on this fact. He is able to construct all of the necessary drawings of a vault from calculations deduced from a single curve. Distances are measured through the use of Euclid's Intercept theorem, and are then plotted into a polar grid in order to obtain the resulting elliptical profile. Similarly, this project constrains variation through three techniques.

The first technique is taken directly from Guarini: the use of the semicircle. In almost all of Guarini's drawings the starting point is the semicircle. Similarly, many of the animations in this project begin with a semicircle and vary its radius over time to produce variation across the projections. This is a limited form of variation, because it is confined to size alone.

The second form of variation has to do with the receiving sections of the projection. All of the animations begin with a projection of one source curve onto the section of a specific geometric object. The sections have been limited to those of cones, cylinders, and tori. However, in the course of an animation the height, width, and location of the sectional profile of a given figure changes. The cone oscillates and scales, changing the angle of intersection and the diameter with each new projection. The cylinder also scales and changes position. The toroidal section changes from an ellipse of such a small minor diameter that it approaches a line, through a circle, and onto a horizontally attenuated ellipse.

The last form of variation introduces a variable conic section as the source curve for all of the projections. An oscillating triangle is intersected by a vertical line producing measurements that are used to orthographically construct the corresponding conic sections. The variable conic section is the source curve for all the subsequent projections, and

while it changes between circle, ellipse, parabola, and hyperbola the curvature is consistently bound to limits of conic section producing variation of an invariant type. This continuity allows the entirety of the drawing process to produce a large set of variable curves that remain measurable through their link with a known source figure.

MINIMAL FIGURES

Guarini's drawings represent each geometric object with as little information needed to describe its form. The cone that is used unroll a torus is represented as a single right triangle, reducing the representation to half of the planar section through vertex of the cone. Similarly, within this project cones are represented as triangles in section and their dimensional properties are extracted through a measuring algorithm that follows the same logic that Guarini used. The major diameter of an elliptical conic section can be found by measuring the total length of the line passing through both sides of a given triangle. The minor diameter can be found by projecting the midpoint of the previous line perpendicularly until it intersects a circle centered on the cone at the same height as the midpoint. The operation and the subsequent algorithm eliminate the need to represent whole objects through constructed "views." Instead the properties of the object are used to find the required measurements without any additional graphic information. This reduction of information focuses both the code and the animation on the representation metric relationships in lieu of visual ones.

PRIMITIVE TEMPLATES

The cone, the sphere, and the cylinder are geometric solids derived from a circle. Within a digital framework a cone is a graphical object that is a built-in component in most software platforms, a primitive. The same could be said for the sphere or the cylinder. However, these singular forms are composed of sets of simpler geometric elements: the circle, the line, and the point. In the history of architectural drawing, the ability to break down the cone, the sphere, and the cylinder into specific geometric properties has made these figures not only significant formal elements but also drawing instruments in their own right. Guarini used cones to unroll toroidal vaults, hemispherical domes, and systematically distort the semi-circle. These three simple solids, the cone, the sphere, and the cylinder can therefore be understood as geometric elements capable of describing forms of a higher degree of complexity than themselves. By extending this logic into the digital realm, it is possible to imagine geometric primitives as two-dimensional drawing instruments. Drawings instruments that are capable not only of creating simulated three-dimensional form, but also describing form through flat two-dimensional orthographic projections.

In "Observation Nine" of chapter three in the fourth tractate, Guarini provides a method of projecting a semicircle onto a cylinder. Referencing Euclid, he bypasses a large amount of projection by providing a template as a drawing device. The drawing device, is the section through the cylinder at the angle of intersection. This template is so specific, that it can be used

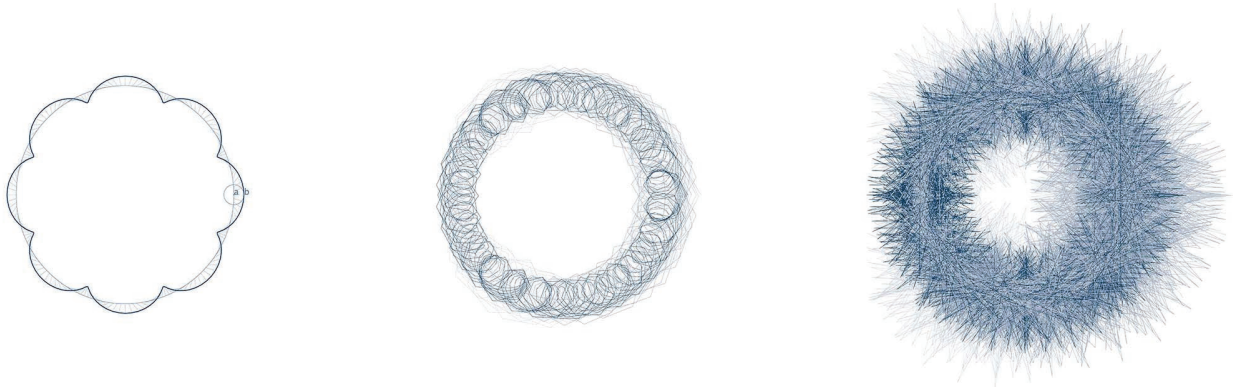


Figure 4. A Baroque Epicycle interrupted through the repetitive addition and subtraction of points. Image by Author.

for no other task. The same principle is used in this project. All of the dimensions extracted from the measuring algorithm in section are then fed into the production of templates in plan that vary in shape and size. All of the templates vary between the limiting figure of either an ellipse or a circle. Whereas Guarini produced a single template, the animations utilize a series of variable templates at each instance of intersection.

INTERRUPTED EPICYCLES

Guarini's drawings do not move, and do not vary over time. However, the end of his treatise contains the drawings of his centrally planned churches. The figure of a centrally planned church can be linked to the kinematic figures of epicycloids and hypocycloids. Epicycloids and hypocycloids are produced by the orbit of once point rotating about another point that is rotating about a fixed center. An epicycloid produces convex curvature, while a hypocycloid produces concave curvature. If we imagine that the centrally planned churches at the end Guarini's treatise are generated by continuously rotating objects, the path of rotation is fixed. The epicycloid or hypocycloid that generates one centrally planned figure continues to generate that figure infinitely without variation. While the figure is created by movement, it is an intentionally limited one. The final state of the figure is the same as its starting state. This project created a procedure to generate variation over time. The motion of epicycloid is no longer limited to completing a fixed number of rotations about a given cycle. Instead the number of rotations change. The epicycloid is interrupted, shifting simultaneously from a cycle of four rotations per revolution to eight or one-hundred. The interruptions are generated by simple arithmetic, maintaining a link with its 17th century precedent (figure 4).

Furthermore, because all regular polygons can be derived from a circle, each incidence of rotation shifts between varying degrees of curve approximation. The epicycloid

instead of being a fixed entity, transforms into a movement that describes figures oscillating between the curvilinear and the polygonal, the multiple and the singular. All of the animations move along the path of an interrupted epicycle. The orthographic projection instead of being a fixed and known quantity shifts and follows the path of constantly changing epicycle.

REVERSIBLE PROJECTIONS

Two-dimensional orthographic projection operates by describing any object with two sets of information in which all of the objects measurements are accounted for. This does not mean that each set of information must correspond to a visual representation of the entire object, it needs only to provide all metric information necessary to describe that object. The use of the semicircle in stereotomy has been noted by a number of scholars as central not only for structural reasons, but also for reasons of reversibility. In almost of all of the examples in "Dell' Ortografia Gettata," Guarini begins with the semicircle. This allows him to work in reverse towards the original planar figure of the semicircle when solving problems of measurement. A key example of this, is that of barrel vault generated by projecting a semicircle onto a cone. In this instance Guarini begins by unrolling the original semicircle to obtain the unrolled dimensions of the complex curves of intersection. Because the semicircle is connected to the distorted circle with parallel lines, unrolling the original semicircle results in unrolling the more complex distorted curve.

This same process is built into the animation. Each projection is unrolled from its elevation by cross referencing the heights of the original semicircle or conic section with the horizontal information in the elevations. The dimensions of the original semicircle are unrolled upon a line in conjunction with all of the irregular curves generated by a given instance of projection (figure 5).

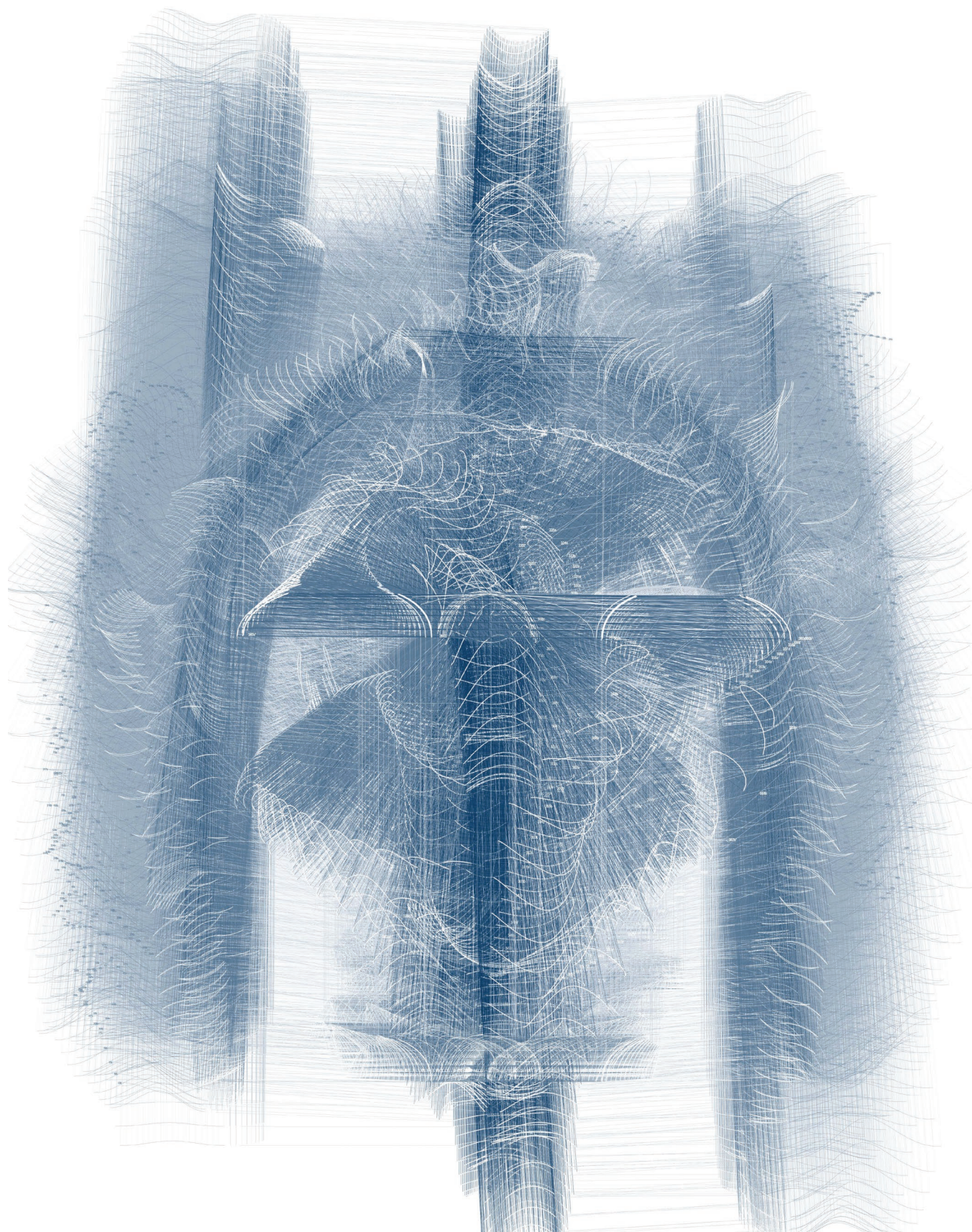


Figure 5. The two dimensional orthographic projection of a semi-circle onto an interrupted epicycle of Tori. Image by Author.

Reversibility is only useful if the starting point is simpler than the result. It creates a direct link between a known and measurable source figure with an unknown and nameless curve. It was not only practical for Guarini, but it also allowed him to explore a wide range of forms, while still maintaining conceptual link to a known and accepted geometric figure, the circle.

DORMANT GEOMETRY

Orthographic projection is a diverse set of procedures that have been grouped under a series of different names for the single purpose of describing the metric properties of architecture by the most expedient means possible. Prior to Gaspard Monge's codification of descriptive geometry in 1795, orthographic projection was composed of information passed down through the practical geometry of mason guilds and the erudite geometry of architectural treatise. Legal and theoretical debates in regards to its purpose and limits are parts of its history. It therefore emerged as loose framework of Euclidean principles that were continually adapted and debated to resolve problems in the metric description of form.

The architect Jane Burry has argued that orthographic projection is "on its way to join the reliable ranks of dead and dormant geometries." This is a reference to Robin Evans's description of orthographic projection. Evans argued that a dead geometry is a geometry whose fundamental tenants are no longer the subject of debate. They have been proven beyond doubt and are therefore more useful to the architect because they are an "inoculation against uncertainty." Orthographic projection's principles are based on Euclid's Elements and in Poincaré's terms are "ordinary." It is the fact that they are ordinary that makes them so useful in describing things far more complex than themselves. It is these ordinary principles and not their materialization in plan, section, and elevation that continue to have relevance in the description of architectural form.

While orthographic projection is largely understood as a representational practice tied to the production of views, the study of the development of its technical procedures presents an alternate history—a history in which the representation of form was secondary to the production of its measure. The drawings produced by these procedures (such as Guarini's) are graphically complex and do not directly correspond to a views of an object. They present measure and calculation not only as process, but as outcome. They demonstrate a line of architectural inquiry in which computation of form is prioritized over its representation. They present trajectory for architectural drawing in which in the description of form is independent of its visualization.

ENDNOTES

- 1 Leon Battista Alberti, *The Art of Building in Ten Books*, trans., Joseph Rykwert, Neal Leach, and Robert Tavenor (Cambridge, MA: The MIT Press, 1988).
- 2 Despite John May's insightful article on the state of orthography in architecture, his critique is tied solely to production of orthographic views, and does not engage the other techniques that are not associated with this practice. See John May, "Everything Is Already an Image," *Log 40* (2017): 9–26.
- 3 David Eberly and Philip Schneider, *Geometric Tools for Computer Graphics* (San Francisco: Elsevier Science, 2003), 159-160.
- 4 Jane Burry, "The Shifting Ground of Architectural Geometry: Getting to Know the New Representational Space," *Architectural Theory Review* 15, nos. 2–10 (2010): 187-200.
- 5 Giuseppe Fallacara, "Digital Stereotomy and Topological Transformations: Reasoning about Shape Building" in *Proceedings of the Second International Congress on Construction History, Volume 1* (Exeter: Short Run Press, 2006), 1075-1092.
- 6 Guarini refers to stereotomy as orthographic projection. See Guarino Guarini, *Architettura civile* (Turin, Italy: Gianfrancesco Mairesse, 1737), 191.
- 7 Joel Sakarovitch, "Stereotomy, A Multifaceted Technique," in *Proceedings of the First International Congress on Construction History* (Madrid: Instituto Juan de Herrera, Escuela Técnica Superior de Arquitectura, 2003).69-79.
- 8 Robin Evans, *The Projective Cast: Architecture and Its Three Geometries* (Cambridge, MA: The MIT Press, 1995).
- 9 Robert Willis, *On the Construction of Vaults in the Middle Ages* (London: RIBA, 1910), 1-2.
- 10 Evans, *The Projective Cast: Architecture and Its Three Geometries*.
- 11 Werner Müller, "The Authenticity of Guarini's Stereotomy in His 'Architettura Civile,'" *Journal of the Society of Architectural Historians* 27, no. 3 (1968): 202–8. <https://doi.org/10.2307/988502>.
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- 16 J.V. Field and J.J. Gray, *The Geometrical Work of Girard Desargues* (New York: Springer Verlag, 1987).
- 17 Müller, "The Authenticity of Guarini's Stereotomy in His 'Architettura Civile.'"
- 18 Guarini, *Architettura civile*, 205-207.
- 19 George L. Hersey, *Architecture and Geometry in The Age of the Baroque* (Chicago: University of Chicago Press, 2000), 183-201.
- 20 Robin Evans, "Translations from Drawing to Buildings," in *Translations from Drawing to Building* (Cambridge, MA: The MIT Press, 1997), 153–89.
- 21 Jacqueline Gargus, "Guarino Guarini: Geometrical Transformation and the Invention of New Architectural Meaning," *Harvard Architectural Review* 7 (1989): 116–31.
- 22 Sakarovitch, "Stereotomy, A Multifaceted Technique."
- 23 Burry, "The Shifting Ground of Architectural Geometry," 199-200.
- 24 Evans, *The Projective Cast: Architecture and Its Three Geometries*, xxvii.
- 25 Henri Poincaré, *Science and Hypothesis* (New York: The Science Press, 1913).